DYNAMICS AND KINEMATICS OF PARTICLES

A key thing in dynamics is that the body already has acceleration

Three things can occur when force is applied to a body

1. Motion (The body can be placed)

2. Deformation (Compression or elongation of the object)

3. It can break

Dynamics can be divided into two namley:

Kinematics: The study of the geometry of motion. Kinematics is used to relate displacement. Velocity and acceleration and time, without reference to the cause of the motion. Kinetics can therefore be simply defined as the study of motion.

Kinetics: The study of the relation existing between the forces acting on a body, the mass of the body and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

In Kinetics we will be talking about **rectilinear motion**, and **curvilinear motion**.

Rectilinear motion can have some of these properties

1. Constant acceleration

2. Constant or uniform velocity

3. Non-constant acceleration

Uniform motion → Constant velocity

Uniformly accelerated motion → Constant acceleration

For a not constant acceleration, it can be solved with

1. Equations

2. Graphical solutions

Curvilinear motion

This involves calculating motion parameters around curves

Circular motion

Polar coordinate

Cylindrical coordinate

KINETICS

This involves trying to relate force and motion. There are four ways that this can be done

1. Inertia

2. Energy and work done method

3. Impulse method (Law of conservation of linear momentum)

4. Law of conservation of energy

TERMS USED IN KINEMATICS

1. Position: This is usually denoted as **X** and it is defined as the location of an object in space at an instance. When we say space we are talking about planes (e.g. Cartesian planes). The 2. Initial position of a particle: Denoted as , is defined as the position of the particle at start of motion. It is **constant** and therefore never changes

3. Displacement: This is defined as the change in position of a particle. It is a vector quantity and therefore pays attention to direction.

Displacement = final position – initial position (Irrespective of the path)

That means if you start from a position and come back to that position, you have no displacement even if you may have covered a distance

Displacement D = {X rsub B} – {X rsub A}

4. Distance: This is the change in position of a body in one direction. When calculating the total distance covered, the method I use is to calculate is to add the distances between two positions till the particular time. For example, if we are to calculate the total distance covered by 4 seconds. We calculate the positions at each time from t=0 to that point. Then we find the gap (absolute value of the displacement) between each time i.e. the distance between t=0 and t=1, gap between t=1 and t=2 etc. After that, we add the values together to get the total distance covered.

5. Average velocity: This pays attention to distance. It basically means in one second, how much distance you can cover. The average velocity of a particle over the time interval %delta t is defined as the quotient displacement and the time interval

The SI units for x and t are metres and seconds respectively

The US customary units are feet and seconds respectively

6.Instantaneous velocity. This is defined as the velocity at an instance. To find this velocity, we want our change in time to be equal to zero

7. Initial velocity is the velocity at the start of the journey denoted by . It is also **constant**.

The velocity v – known as the speed of the particle – can be positive or negative. It is positive when x is positive and vice-versa.

8. Acceleration: This is by how much you can increase your speed in a second.

9. Average acceleration: This is the time rate of change of velocity

10. Similarly to instantaneous acceleration

Even though acceleration and velocity are vectors, since we are dealing with rectilinear motion where the particle has a known and fixed direction, we only need to specify the sense (+ve or -ve) and the magnitude (scalar quantity).

Acceleration can be positive or negative. A positive value indicates that velocity increases. This may mean that the particle is moving faster in the positive direction or moving slowly in the negative direction. In both cases delta v is positive.

11. Deceleration: The term deceleration is used to refer to acceleration when the speed of the particle decreases; the particle is then moving more slowly.

In physics or sciences when a big object is considered a particle, that means calculation done on that object is done without regard to its size.

We will look at uniform motion and uniform accelerated motion.

A particle moving along a straight line is said to be in **rectilinear motion**. At a given time t, the particle will occupy a position on the straight line – usually with respect to an origin –. The position/distance can either be positive or negative depending on the position relative to the origin. So if it is on the left, then it is negative. However, if the position is on the right of the origin, then it is positive.

When the position coordinate x of a particle is known for every time t, we say that the motion of the particle is known. The timetable of the motion can be given in the form of an equation in x and t, such as x = 6t^2 – t^3, or in the form of a graph of x versus t.

Examples:

Consider a particle moving in a straight line and assume that its position is defined by the equation

x = 6t^2 – t^3

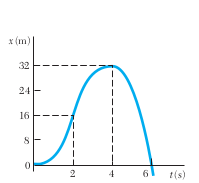
where t is expressed in seconds and x in metres. The instantaneous velocity – velocity at any time t – is obtained by differentiating x with respect to t

v = dx/dt = 12t – 3t^2

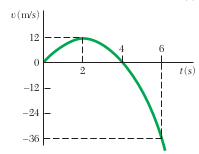
Acceleration a, is obtained by differentiating velocity.

A = dv/dt = 12 – 6t

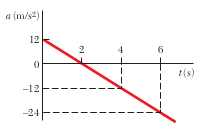
Note the following **motion curves**.



Displacement-time graph



Velocity time graph



A study of the three motion curves of Fig 11.6 shows that the motion of the particle from t=0 to t=infinite can be divided into 4 phases

1. The particle starts from the origin x=0, with no velocity but with a positive acceleration. Under this acceleration, the particle gains a positive velocity and moves in the positive direction. From t =0 to t = 2s, x, v and a are all positive

2. At t=2s, the acceleration is zero; the velocity has reached its maximum value. From t = 2 to t = 4, v is positive but a is negative; the particles moves in the positive direction but reduces in velocity.

3. At t = 4s, the velocity is zero; the position coordinate x has reached its maximum value. From then on, both v and a are negative; the particle is accelerating and moves in the negative direction with increasing speed.

4. At t=6s, the particle passes through the origin; its coordinate x is then zero while the total distance traveled since the beginning of the motion is 64m. For values of t larger than 6s, x, v and a will all be negative. The particle keeps moving in the negative direction away from O faster and faster

DETERMINATION OF THE MOTION OF A PARTICLE

When => Uniform motion

When => Uniform accelerated motion

Motion is not always defined by a relation between x and t. Acceleration can be expressed in different forms and depending on what causes the acceleration. The acceleration of the motion of a body tied to a spring will depend on factors relating to the elongation of the spring.

v = dx/dt

dt = dx/v

a = dv/dt

a = dv div dx/v

a = v dv/dx

Three common classes of motion

1. When acceleration is given as a function of time, f(t). We will like to have an integral of that function with respect to time.

Similarly for the velocity

v\_o and x\_o are the initial velocity and initial position respectively. They are the initial conditions of the motion

2. When acceleration is given as a function of x f(x)

For velocity

3. As a function of v

f(v) = dv over dt

f(v) = v dv/dx

dt = dv/f(v)

dx = vdv/f(v)

UNIFORM RECTILINEAR MOTION

This is motion where there is no acceleration and velocity is constant

UNIFORM ACCELERATED RECTILINEAR MOTION.

Here, acceleration is constant

**QUESTIONS**

1. The position of a particle which moves along a straight line is defined by the relation.

, where x is expressed in feet and t in seconds. Determine:

a. The time at which the velocity will be zero. Answer: t=5

b. The position and distance traveled by the particle at that. Answer: position = -60ft, distance traveled = 100ft

c. The acceleration of the particle at that time. +18

d. Distance traveled by the particle from t=4 to t=6. Answer: 18ft

2. A ball is tossed with a velocity of 10m/s directed vertically upward from a window located 20m above the ground. Knowing that the acceleration of the ball is constant an

a. the velocity v and elevation y of the ball above the ground at any time t. Ans: v=10-9.81t, y=20+10t-4.905t^2

b. The highest elevation reached by the ball and the corresponding value: Ans: t=1.019s y=25.1m

c. The time when the ball will hit the ground and the corresponding velocity: t=3.28s, v=-22.2

d. Draw the v-t and y-t curves

3. The brake mechanism used to reduce recoil in certain types of guns consists

essentially of a piston attached to the barrel and moving in a fixed cylinder

filled with oil. As the barrel recoils with an initial velocity v0, the piston

moves and oil is forced through orifices in the piston, causing the piston

and the barrel to decelerate at a rate proportional to their velocity; that is,

a 5 2kv. Express

(a) v in terms of t,

(b) x in terms of t,

(c) v in terms of x.

d. Draw the corresponding motion curves.

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4. The acceleration of a particle is directly proportional to the square of the time t. When t = 0, the particle is at x = 24 m. Knowing that at t = 6 s, x = 96 m and v = 18 m/s, express x and v in terms of t.

If you look at the question, you’ll see that we were given a few values.

On integrating, you’ll s get the value of the position

int from{24} to{x} {v}

The reason that we used the value when t=0, is so that we can get the equation for x

3v-54 = {} - 216k

Looking at the values they gave us in the question, we can start that as our base values in the integration